Deciphering the oscillation spectrum of γ Doradus and SPB stars

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Context

γ Doradus (late A-early F type) and Slowly Pulsating B-type (SPB) stars pulsate in high radial order gravity modes that probe the deep interior structure at the boundary between the convective core and the radiative envelope. Their seismic study could potentially give observational constraints on convective boundary mixing (Miglio et al. 2008) and angular momentum transport (Bouabid et al. 2013) that are still major uncertainties of stellar models. Nonetheless, these stars also rotate quite fastly to the extent that mode identification is particularly challenging. Classical perturbative methods are indeed insufficient to describe the rotation-pulsation coupling accurately enough in most cases.

With the intent of solving this problem, we have developed a model-independent method to disentangle the oscillation spectrum of rapidly-rotating γ Dor and SPB stars that is based on the traditional approximation of rotation.

The method

In the case of a rotating star, solving the basic equations of stellar pulsations is a full 2D problem that is both numerically complex and computationally expensive. The traditional approximation of rotation (TAR) treats the rotation-pulsation coupling in a simplified manner in order to get back to a 1D set of equations while still accounting for the main effects of the Coriolis force on the pulsations. Although the hypotheses of the TAR are quite severe (no centrifugal distortion, solid body rotation), several studies (e.g. Ballot et al. 2012) suggest that it is still a good approximation to describe the properties of gravity modes in γ Dor and SPB stars.

In the asymptotic formulation of the TAR, the pulsation periods of high radial order g-modes follow a semi-analytic formula in the co-rotating frame of reference,

$$\boxed{P_{\mathrm{n},\ell,\mathrm{m}}\left(s\right) \approx \frac{P_0\left(n+\epsilon\right)}{\sqrt{\lambda_{\ell,\mathrm{m}}\left(s\right)}}}$$

n radial order
ℓ angular degree
m azimuthal order
ϵ near constant dependent on the star's structure

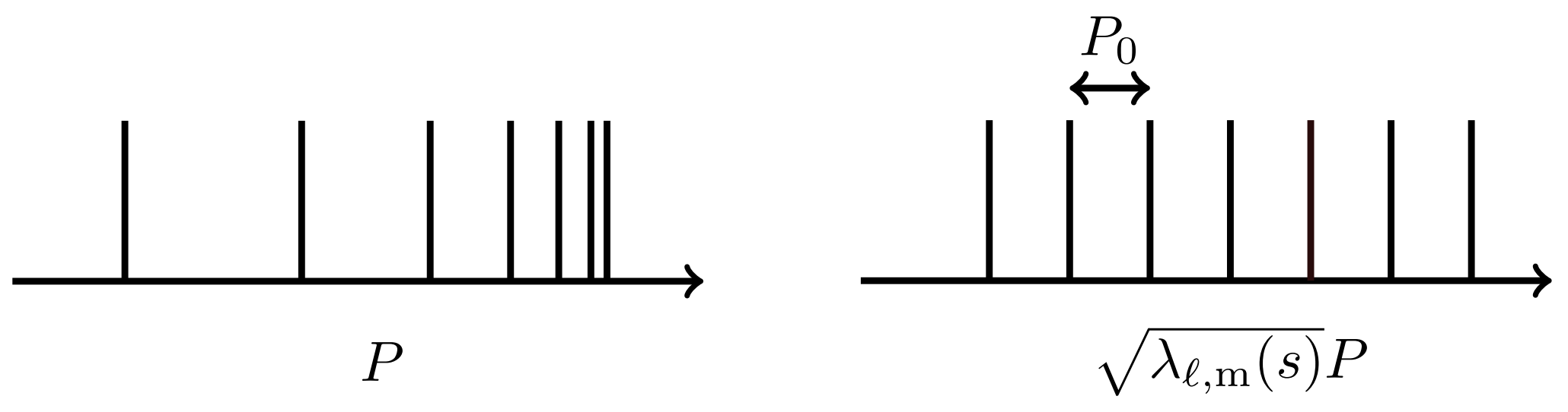
where P_0 is the buoyancy radius

$$P_0 = 2\pi^2\left(\int_{\mathcal{R}} \frac{N_{\mathrm{BV}}}{r}\mathrm{d}r\right)^{-1}$$

N_{BV} Brunt-Väisälä frequency
$\mathcal{R}$ resonant cavity

and $\lambda_{\ell,\mathrm{m}}$ are the eigenvalues of Laplace's tidal equation, which depend on:
- the geometry of the pulsation modes (ℓ, m),
- the spin parameter $s = 2\nu_{\mathrm{rot}}P_{\mathrm{n},\ell,\mathrm{m}}$, where ν_{rot} is the rotation frequency.

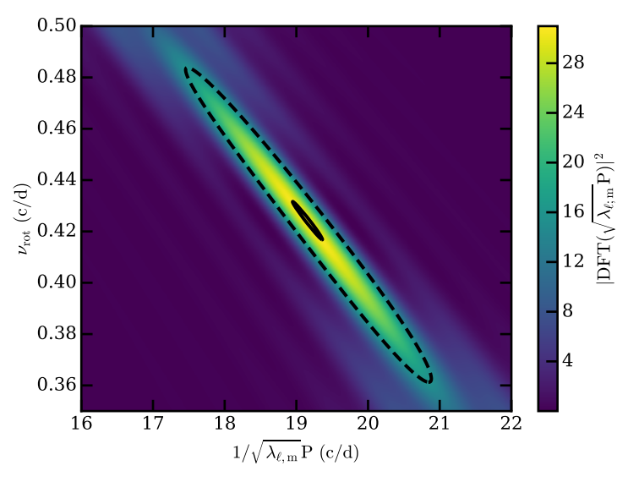
The asymptotic TAR predicts regular features of the g-mode spectrum that can be highlighted by stretching the period axis scale from P to $\sqrt{\lambda_{\ell,\mathrm{m}}\left(s\right)}P$.



We search for these regularities to identifiy modes and determine ν_{rot} and P_0:

1. Extract peak frequencies from the oscillation spectrum
2. Assume a geometry (ℓ, m) and choose a range of ν_{rot} to test
3. For each value of ν_{rot},
   a) Stretch the oscillation spectrum
   b) Compute its Discrete Fourier Transform (DFT)
4. Stack the DFT spectra by increasing ν_{rot}
5. Check if the maximum of Power Spectral Density (PSD) is significant.
   Yes. Output: $(\nu_{\mathrm{rot}}, P_0)$ and mode ID
   No. Iterate → Get back to step 2

Map of the parameter space explored



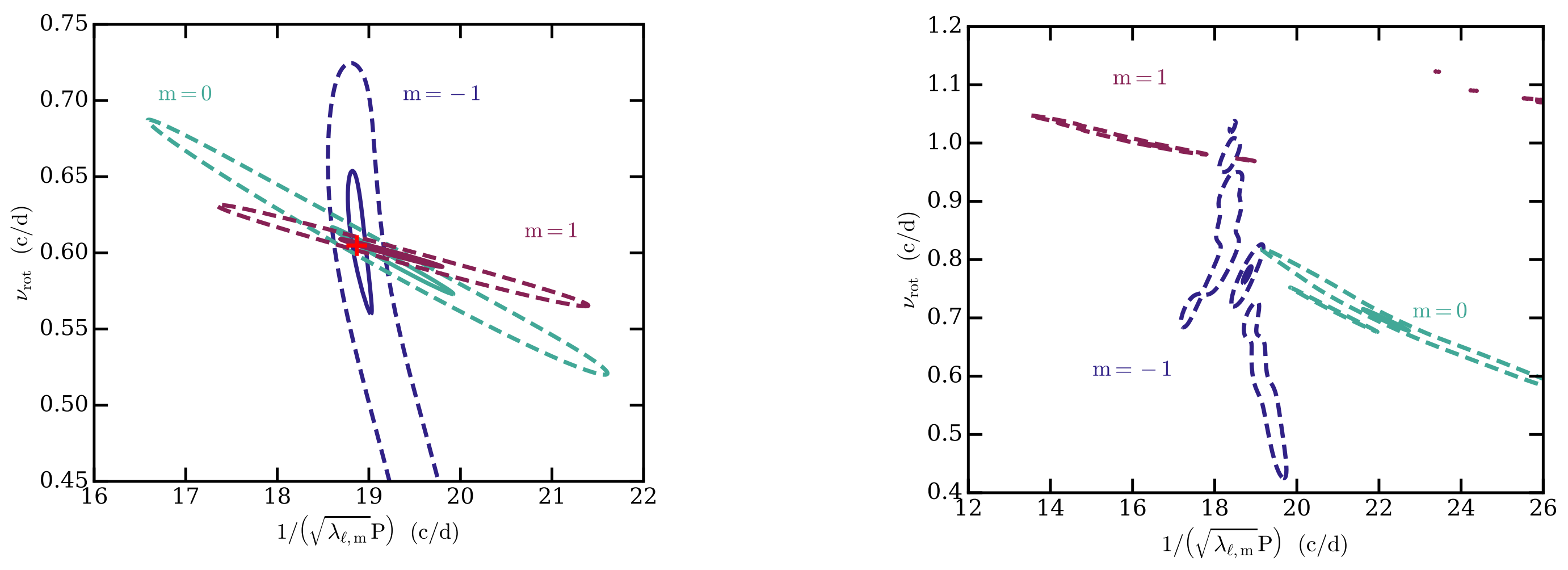
— Contour at 95%
--- Contour at 50%
of the maximum of PSD

Tests on modelled spectra

We computed two spectra of a typical γ Dor model ($\ell = 1$) using the ACOR oscillation code (Ouazzani et al. 2012) that solves the full 2D problem numerically:

a) Solid-body rotation

b) Differential rotation
$1.3\,\mathrm{c/d}$ in the core
$0.6\,\mathrm{c/d}$ in the envelope

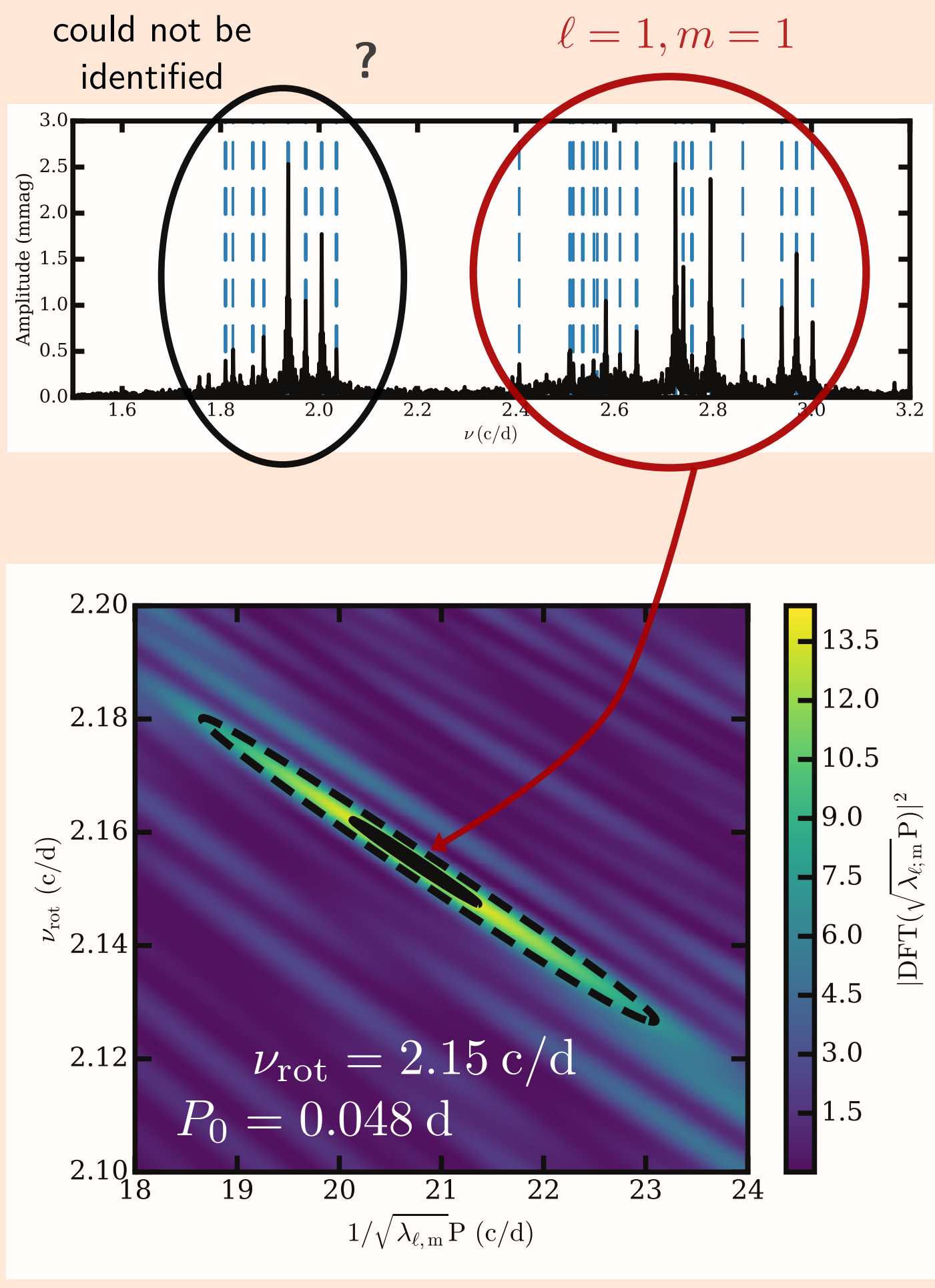


We recovered the model parameters perfectly well for the model in solid-body rotation. In the case of the differentially rotating model, we could detect differential rotation but we could not estimate the parameters reliably.

Results on Kepler targets

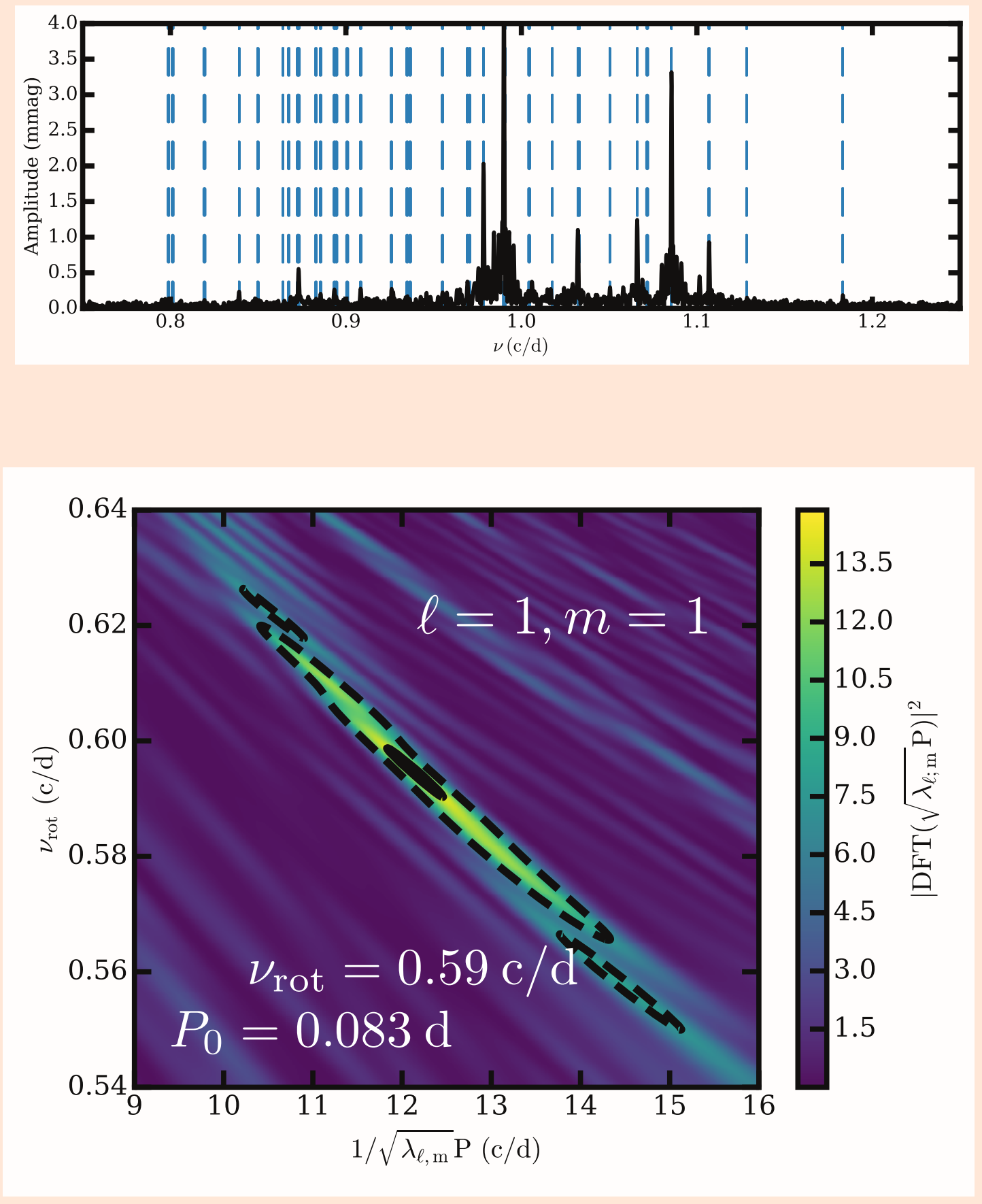
We applied our method on the oscillation spectra of two Kepler targets, the γ Dor star KIC 12066947 and the SPB star KIC 3459497. Both were modelled in the literature (Van Reeth et al. 2016, Pápics et al. 2017) with results consistent with our determination.

The γ Dor star KIC 12066947



Van Reeth et al.: $\nu_{\mathrm{rot}} = 2.160 \pm 0.008\,\mathrm{c/d}$
$P_0 = 0.0484 \pm 0.0012\,\mathrm{d}$

The SPB star KIC 3459497



Pápics et al.: $\nu_{\mathrm{rot}} = 0.63 \pm 0.04\,\mathrm{c/d}$
$P_0 = 0.096 \pm 0.014\,\mathrm{d}$

In short

- A stellar model-independent method to estimate the near-core rotation rate, the buoyancy radius and mode ID in γ Dor and SPB stars.

- Limitations of the traditional approximation but
  - sensitivity to differential rotation
  - possibility of adapting the method to a better asymptotic theory (see e.g. Prat et al. 2017)

- Possible automation of the method → exploit the wealth of data from Kepler and in the near future, PLATO that will observe in a much wider field.

Further information

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or

Come find me at the conference!

References

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