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Modelling of Convection

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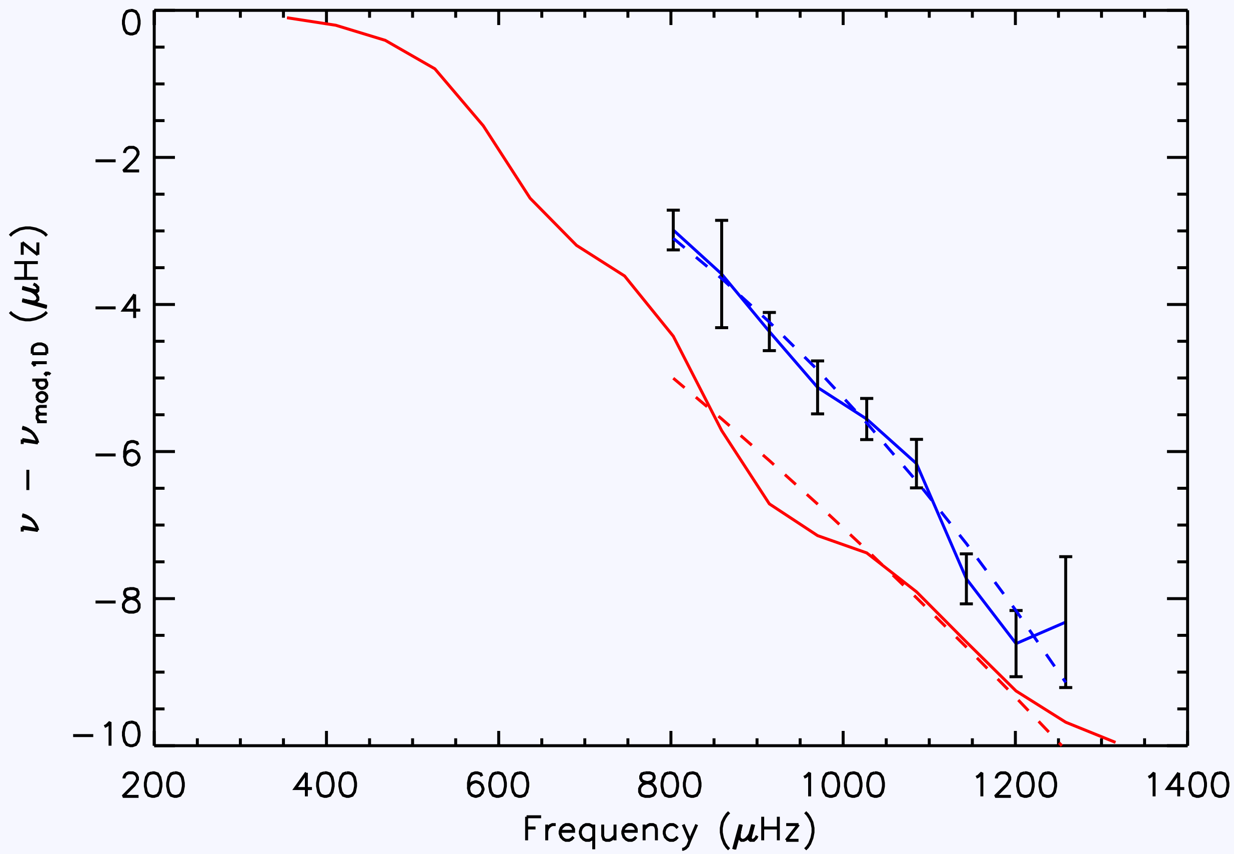
Relevance of Convection I

- **Space asteroseismology and convection**
  - For characterizing planets with PLATO:
    - ✳ asteroseismology → mass M_s, radius R_s, age, luminosity L_s, chemical composition
    - ✳ relative planet masses M_p & radii R_p → absolute ones
    - ➡ exoplanet internal composition & environment
  - For age determination using asteroseismology:
    - ➡ need stellar models
    - ➡ need convection models
    - ✳ available 1D models very incomplete
    - ✳ do-it-all-in-3D-only too expensive
    - ✳ 3D simulations & stellar observations → improve 1D modelling

➡ improving PLATO pipelines for deriving stellar parameters with asteroseismology

Relevance of Convection II

Investigating the Near Surface Effect for HD 49385



3D averaged structure patched from ANTARES simulation compared to standard 1D CESAM model tested by CoRoT observations.

Reported in Kupka, Belkacem, Samadi, Deheuvels (2017), 2nd BRITE Sci. Conf., Proc. Polish Astron. Soc., eds. G. Stachowski, E. Poretti, J. Matthews (in print)

Blue line: difference: seismic analysis (CoRoT data) by Benomar et al. (2009), A&A 506, 15 vs. standard 1D model. 1σ error bars: uncertainties of observed frequencies.

Red line: difference: 3D-based patched model vs. 1D model.

Dashed lines: Kjeldsen et al. (2008) type power law fits.

Relevance of Convection III

- **Physical background**
  - Relevance of convection for stellar physics:
    - ✳ can modify thermal structure of a star → near surface effect on p-modes
    - ✳ can cause / modify p-mode driving / damping → pulsation-convection interaction
    - ✳ mixing, also in nearby “stable” layers → overshooting, element distribution
    - ✳ interaction with rotation & magnetic fields → differential rotation, stellar dynamo, stellar activity
  - Some of these effects also relevant to models of planets

➡ so we want to be able to model convection as accurately as possible !

Continued Need for 1D Models

Table 2 A collection of computable (affordable) and non-computable (unaffordable) 2D and 3D numerical simulations of stellar convection

Problem	N	N_t	C	affordable
3D, solar granules, 6 Mm box	10^8	10^6	1	Yes
3D, the whole solar surface	4.2×10^{12}	10^6	4×10^4	Within 20 years?
3D, turb. granules, 6 Mm box	6.4×10^9	4×10^6	256	becoming
3D, turb. whole solar surface	2.7×10^{14}	4×10^6	1.1×10^7	Non-computable
3D, low res. 200 Mm box	8×10^9	10^6	80	Yes, at the limit
3D, low res. entire c.z.	4.5×10^{12}	5×10^8	2.25×10^7	Non-computable
3D, low res. stellar evolution	4.5×10^{12}	5×10^{18}	2.25×10^{17}	Non-computable
3D, mode damping, entire c.z.	3×10^{14}	5×10^9	1.5×10^{10}	Non-computable
3D, subsurface global solar c.z.	3×10^9	10^5	3	Yes
2D, short period Cepheid, 10°	10^6	10^8	1	Yes
2D, short period Cepheid, 360°	3.6×10^7	10^8	36	Yes, at the limit
3D, short period Cepheid, 360°	10^{12}	10^8	10^5	Within 20 years?

For different problems we compare number of grid points N, number of time steps N_t, computational complexity C relative to the reference problem, and its computability aspects (rounded values for better readability)

The non-computable cases may of course be accessible one day to quantum computers or other, advanced technology. However, the case of $C \sim 1.5 \times 10^{10}$ appears to be too distant for any reasonable prediction and the case of $C \sim 2.25 \times 10^{17}$ remains to be extremely unlikely to be accessible for at least this century

Turb., turbulent; res., resolution; c.z., convection zone

Narrow lines separating perfectly affordable problems for numerical simulations of convection and pulsation in stars from totally unrealistic, unaffordable problems! Table taken from Kupka & Muthsam (2017), Living Rev. Comput. Astrophys. 3:1 (see also for further details).

Criteria for Modelling I

The quest for free parameters

Only way to completely avoid "free parameters": consider highly idealized flow field (→ inadequate for sstellar convection). Examples:

1. Irrotational and Boussinesq: Pasetto et al. (2014), MNRAS 445, 3592
2. Two-point Dirac distribution function (mass flux average: up-/downstream)

Unavoidable parameters even in 3D simulations (3D LES)

1. Mathematical ones: time integration, spatial discretization, ...
2. Viscosity model
3. Boundary conditions

But the key point here is: their necessity itself is not the problem.

What matters: Completeness of included physics & how parameters are adjusted.

Criteria for Modelling II

Criteria

1. Correct physical dimension
2. Invariance of tensor properties and proper behaviour with respect to standard transformations (coordinate systems)
3. Respecting sign- and other symmetries of basic variables
4. Physical and mathematical realizability

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1. Correct physical dimension
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3. Respecting sign- and other symmetries of basic variables
4. Physical and mathematical realizability
5. Robustness: predictions robust with respect to changes of parameters (of few percent?), replacing a model component containing them: robust, too
6. Universality: it should not be necessary to recalibrate the internal parameters for different types of astrophysical objects (e.g., Sun, DA white dwarf, etc.)
7. Computability: formalism must be affordable on present computing means
8. Physical verifiability: the model should allow falsification with observational data or direct numerical simulation or an approach having passed such tests
9. Independence of internal parameters from object being modelled

Criteria for Modelling III

Comparison:	MLT	CM/CGM	non-local	RS	3D LES
1. Dimension:	++	++	++	++	++
2. Invariance:	++	++	+	+	+
3. Symmetries:	+	+	+	+	++
4. Realizability:	++	++	+-	+-	++
5. Robustness:	--	-	-	+- (?)	++
6. Universality:	--	-	+-	+- (?)	++
7. Computability:	++	++	+	+-	-/--
8. Verifiability:	+-	+-	+ (?)	+ (?)	++
9. Object independence:	--	-	+- (?)	?	+

Any grading is inevitably biased, but the intention here is to give an idea about what really is the problem with “free parameters” in classical convection models! (Ideally, an approach would have + or ++ for each criterion.)

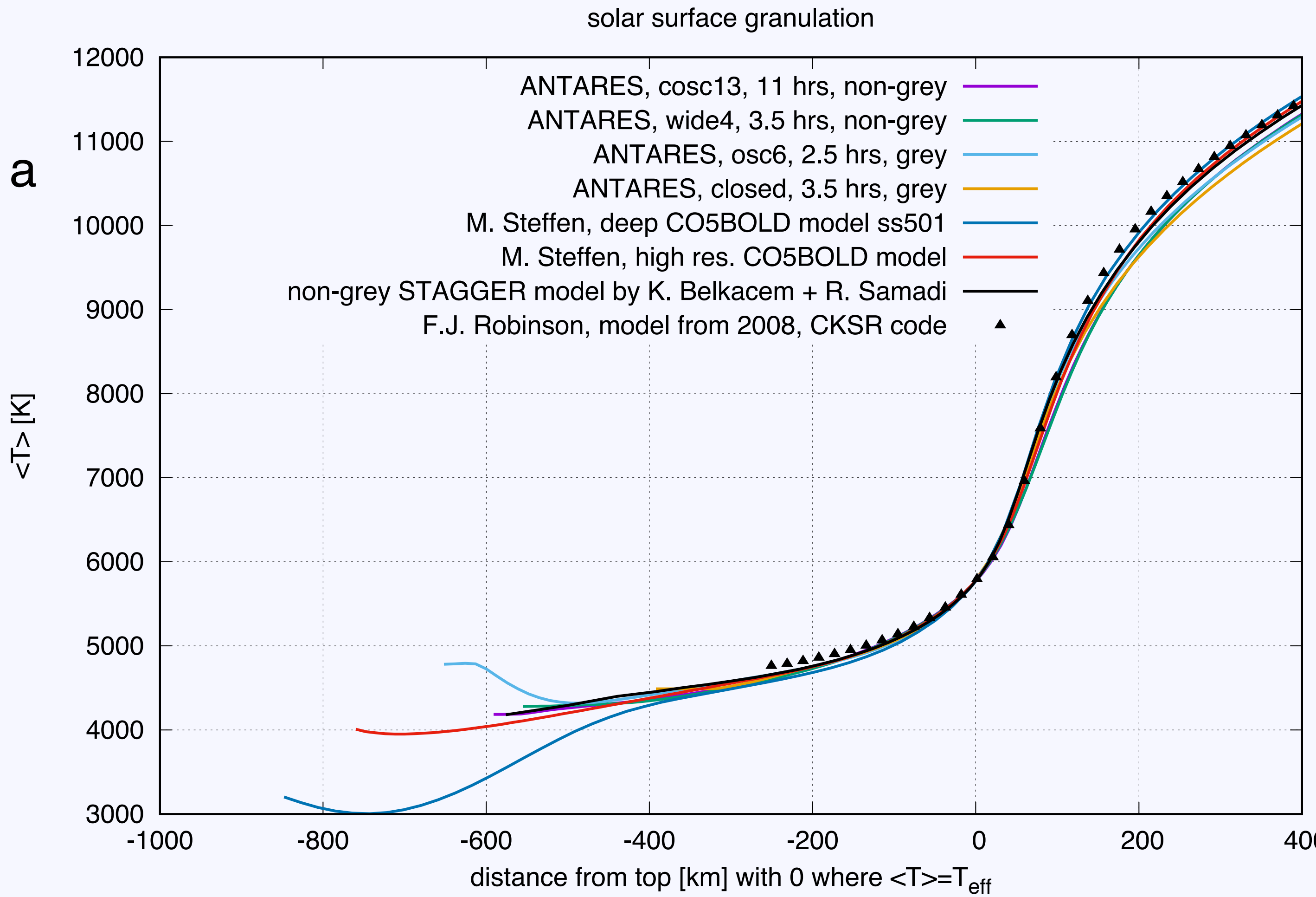
1D+3D Averaged Thermal Structure

Solar surface layers

mean temperature as a function of height

$T_{eff} \sim 5777$ K,
g = 274 m sec^{-2}
(log g = 4.4377),
M = 1 $M_\odot$

3D simulations, averaged horizontally and in time



Guide improvement of stellar models → justifies usage of 3D LES to calibrate convection related effects in model grids → reduces uncertainty in stellar parameter determination

Current Calibrations of 1D Models I

- **Constructing 1D models based on 3D LES**
  - Supported by "universality" of thermal equilibrium structure in 3D LES
  - Calibrate / tune MLT parameter α:
    - reproduce integral property (L) or local target quantity (entropy jump Δs, s_{bot})
  - Scaling laws from 3D LES → entropy as a function of depth, ...
  - Model patching → 3D LES as upper boundary condition

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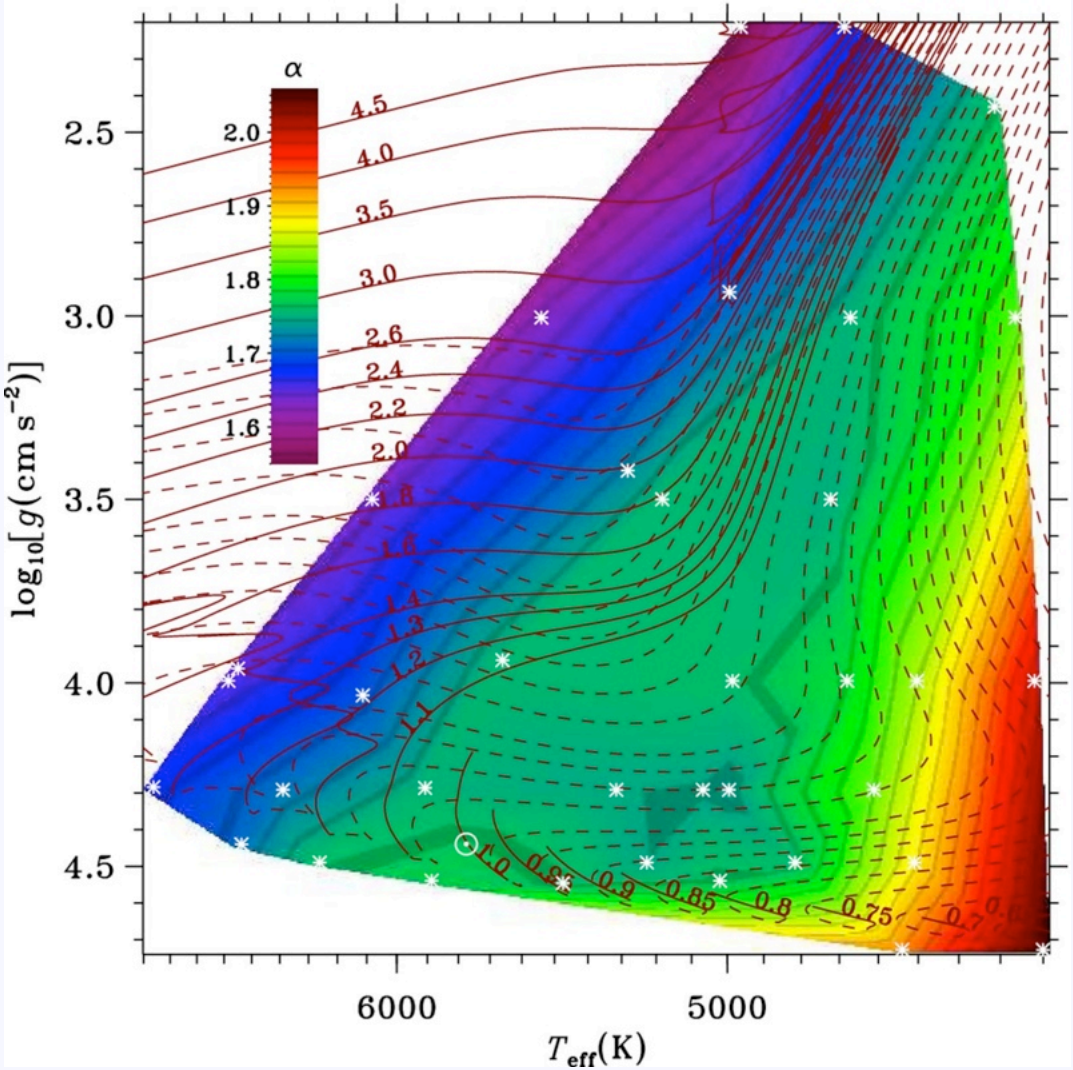
- **Advantages & disadvantages**
  - Calibrate / tune MLT parameter α:
    - most popular: simple, no changes in codes
    - Different properties considered ➔ different values of α, even $\alpha(r)$
  - Scaling laws: "easy"... but accurate ones difficult to derive ...
  - Model patching:
    - results from 3D LES ➔ 1D models. Interpolation in model grids ?

Current Calibrations of 1D Models II

The optimum fit parameter α to reproduce the entropy jump Δs.

The value found for α is sensitive to the exact choice of the dependent variable to be optimized.

Different optimizations hence yield different temperature structures.



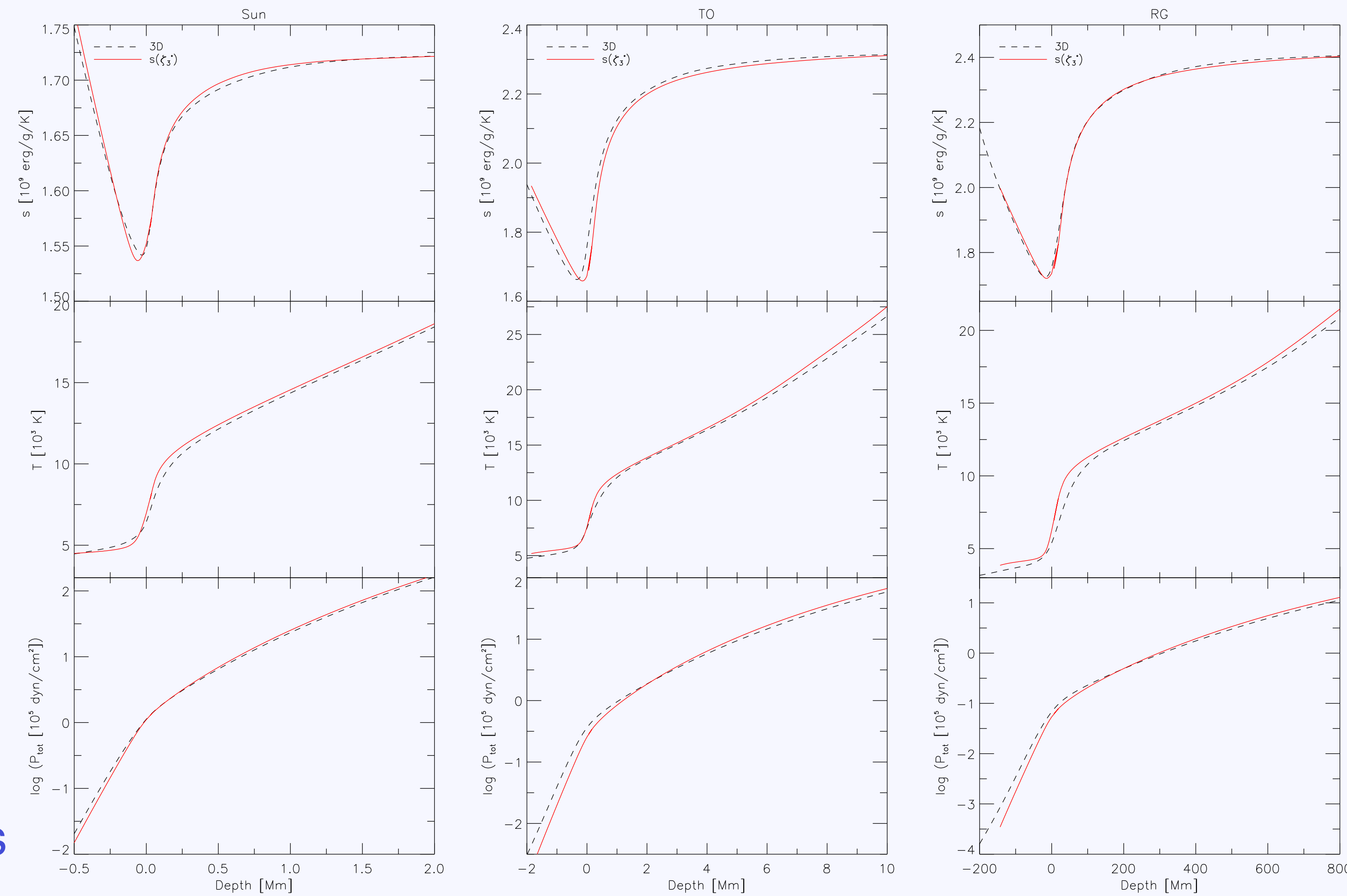
Optimum MLT values α for entropy jump Δs throughout the lower part of the HRD for the STAGGER grid by Trampedach et al. (2014), MNRAS 445, 4366 (Fig. 4).

Current Calibrations of 1D Models III

Entropy as function of depth for the Sun, a the turn off, and in the red giant phase.

Comparison of direct result (black dashes) with scaling formula (solid red line).

This recipe leads to systematic differences in temperature gradients & the pressure structure.



Test of a scaling law for the entropy based on the STAGGER grid by Magic et al. (2013 to 2015) derived by Magic (2016), A&A 586, A88 (Fig. 6).

The Current State

- **Present situation:**
  - The most powerful approach is currently the patching method:
    - thermal structure predicted by 3D LES: ✓
    - 3D LES results carried over as much as possible: ✓
    - Accuracy, if interpolation needed ? Simulation grid density ?
  - Accurate results on velocity fields ➔ boundary conditions !
  - Physically more complete models ➔ not yet target for 3D LES

➡ Take PLATO as an incentive to develop a library of convection models of different complexity (including averaged 3D LES).

A Library of Convection Models I

Facing challenges in convection modelling

- Parameter calibrations of simple 1D models from 3D LES or data
  - ➡ Robustness ?
  - ➡ Universality ?
  - ➡ Object independence ?
- Consistency of model usage ?
  - Stellar evolution → stellar model grids → stellar parameters
  - Asteroseismology → stellar parameters → PLATO data products
  - Planets research → structure, cooling, ...
  - Each with different modelling approaches ?
- In current stellar / planet modelling:
  - Case by case solutions
  - Traditional models dominate (with tests based on specific use cases)
  - Hope that "3D simulations will fix it"

A Library of Convection Models II

Responding to challenges in convection modelling

- ➡ proposal: develop a library of convection models with information on
  - use cases (stars, planets, ...)
  - model grading model & error estimates (test cases),
  - physical/math. assumptions made, ...
- ➡ goal: allow a more systematic approach to
  - estimates of modelling uncertainties
  - avoid/reduce inconsistencies in modelling from adhoc solutions
  - provide guidance in choosing a model
- ➡ Collaborative effort: science development done parallel to PLATO related mission preparation work
  (contacts: F. Kupka @ IAG, G. Wuchterl @ Inst. Astrophys., U. of Vienna)
- ➡ Example for feasibility and advantages ?

Exchange with Geophysics I

Satellite observation of convective boundary layer

Hartmann et al. (1997), Boundary-Layer Meteorology 84, 45-65

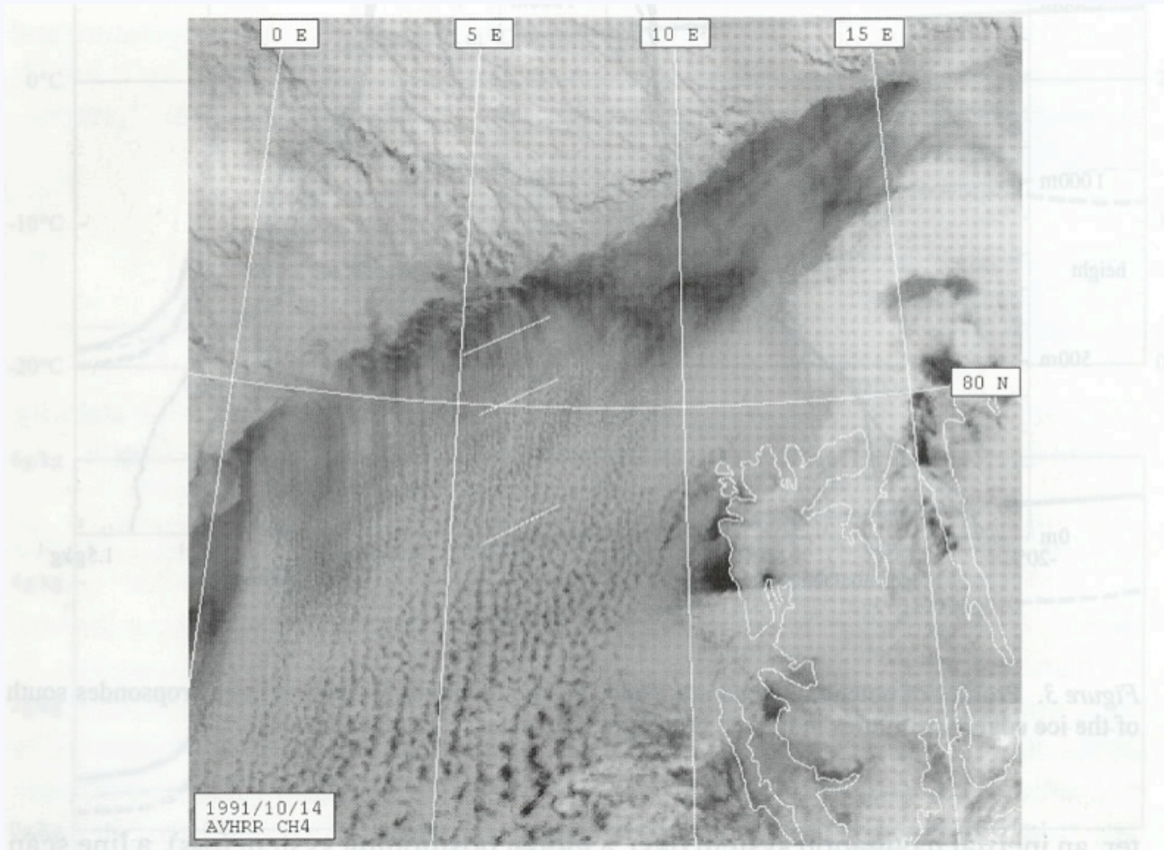
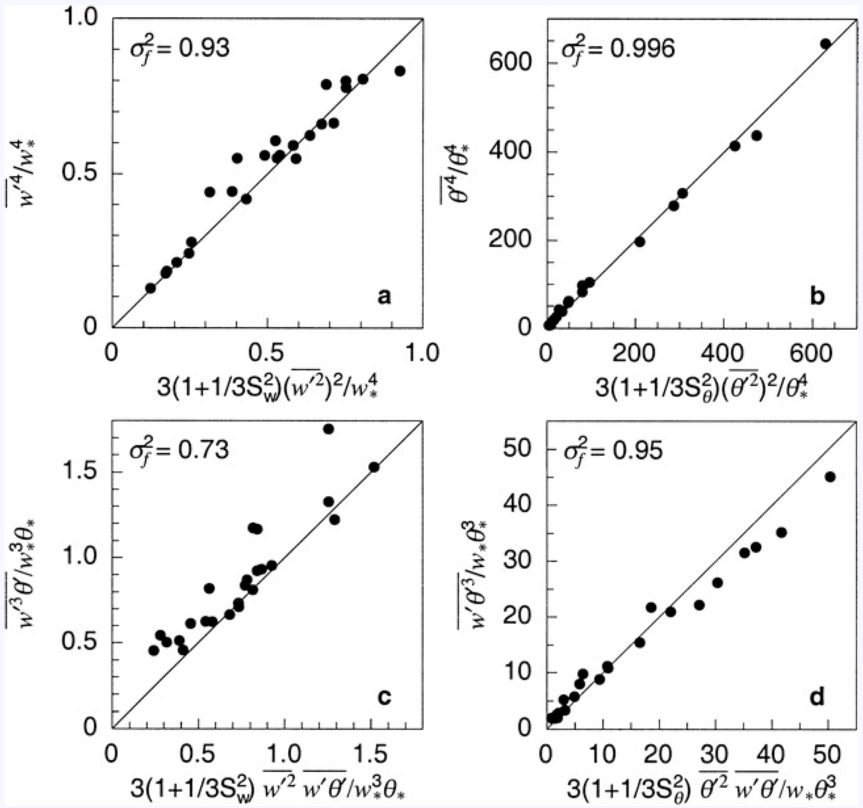
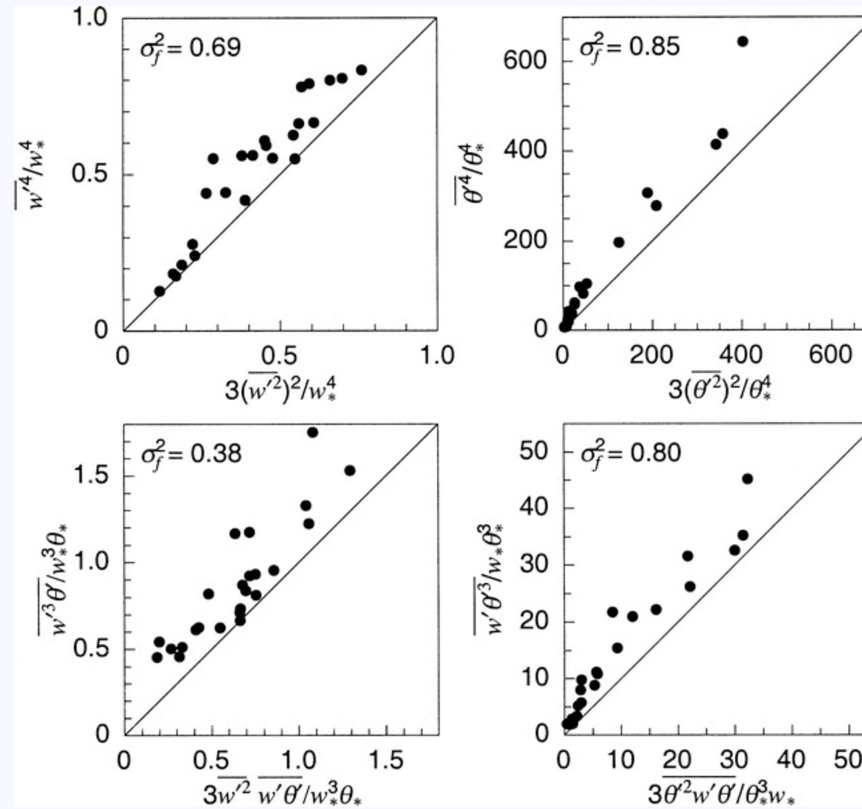


Figure 2. NOAA 11 (channel 4, at 11 UTC) satellite picture of the experimental region showing sea ice in the upper part and Svalbard in the lower right corner. Cloud bands south of the ice indicate vortex rolls. White lines mark the tracks of our three cross-flow aircraft runs.

Exchange with Geophysics II





Normalized 4th order moments of velocity and temperature (ordinate) compared to the model by Gryanik & Hartmann (2002), J. Atmos. Sci. 59, 2729 (left panel) and to the quasi-normal approximation (right panel). Data: in situ measurements from an aircraft within the ARTIST campaign (cf. Hartmann et al. 1997, Bound. Lay. Met. 84, 45).

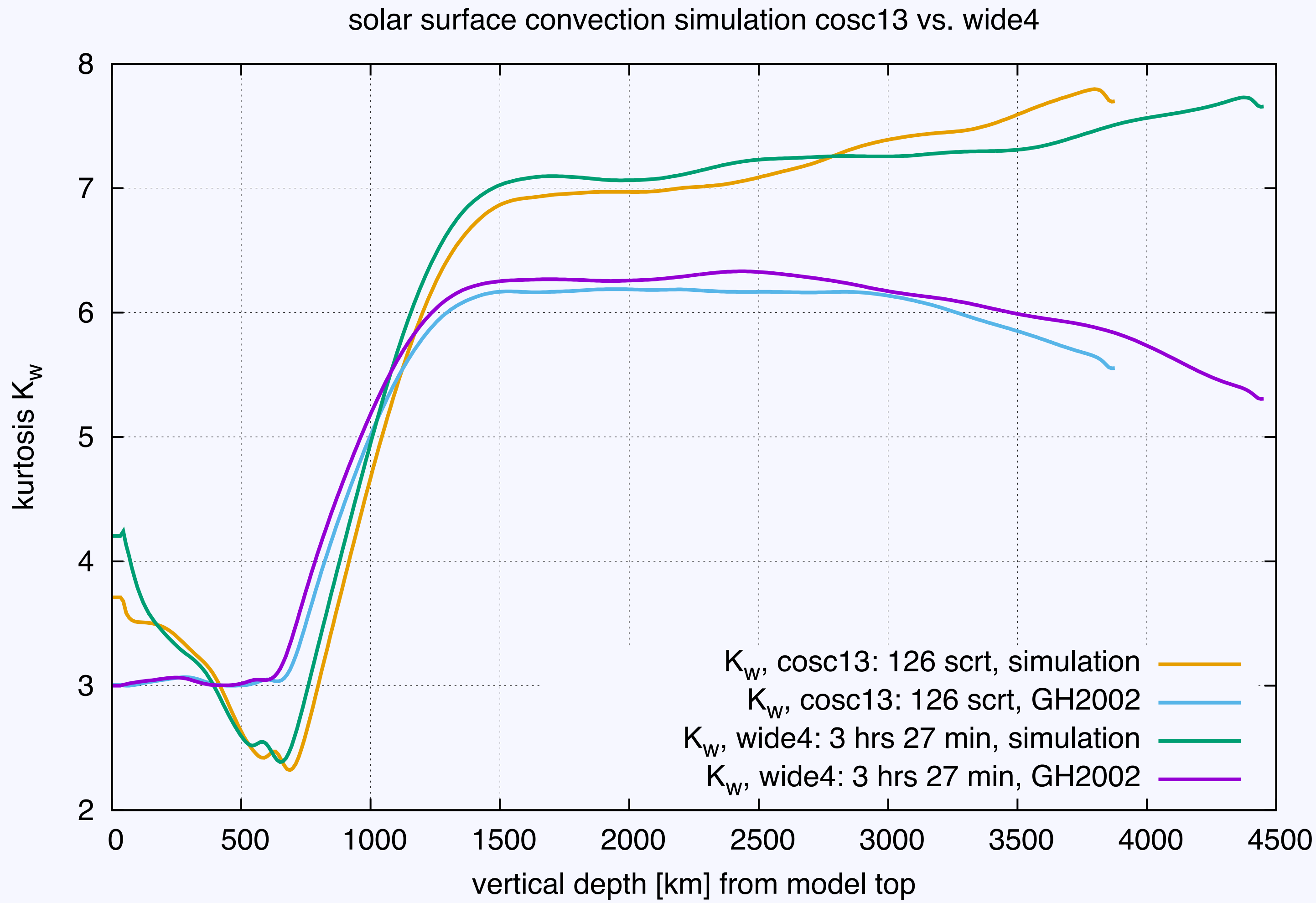
Application to the Solar Case I

Distribution of kurtosis of vertical kinetic energy in granulation simulations

Related quantities are needed in semi-analytical models of p-mode driving

$T_{eff} \sim 5777$ K, g=274 m sec^{-2} (log g = 4.4377),

M = 1 $M_\odot$



Direct computation vs. closure. Note: quasi-normal approximation yields K_w=3.

Application to the Solar Case II

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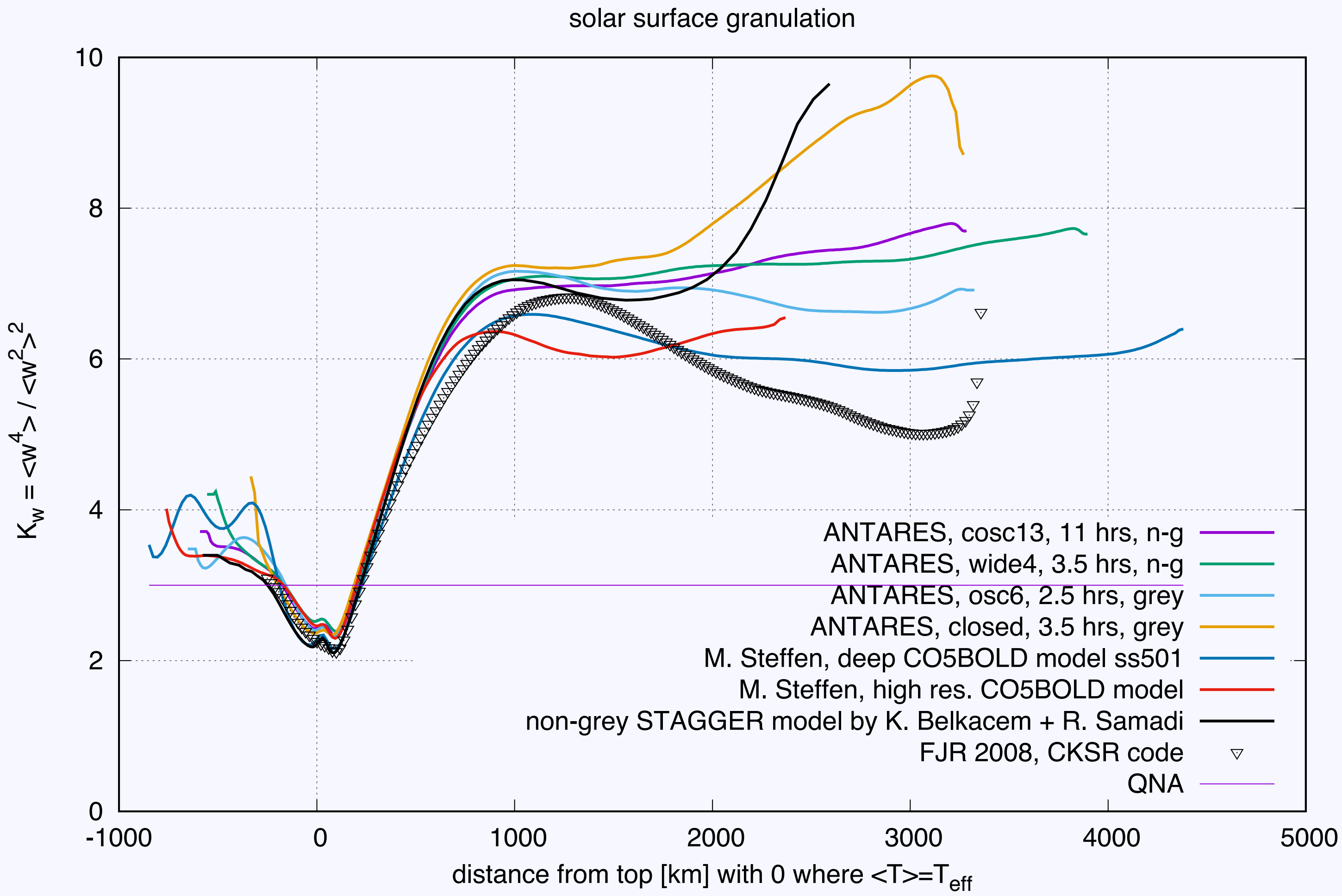
3D simulations: code comparison



Agreement only within and just underneath the superadiabatic layer. Within a region of up to three pressure scale heights very sensitive to boundary conditions (esp. lower ones).

Application to the Solar Case III

solar p-mode excitation rates

using the "Closure Model with Plumes", CMP, K. Belkacem, R. Samadi, M.-J. Goupil, F. Kupka, A&A 460, 173 (2006), based on the GH model

CMP vs. QNA and LF vs. GF illustrate the non-Gaussian behaviour of solar convection (dots: solar data)

K. Belkacem, R. Samadi, M.-J. Goupil, F. Kupka, F. Baudin, A&A 460, 183 (2006)

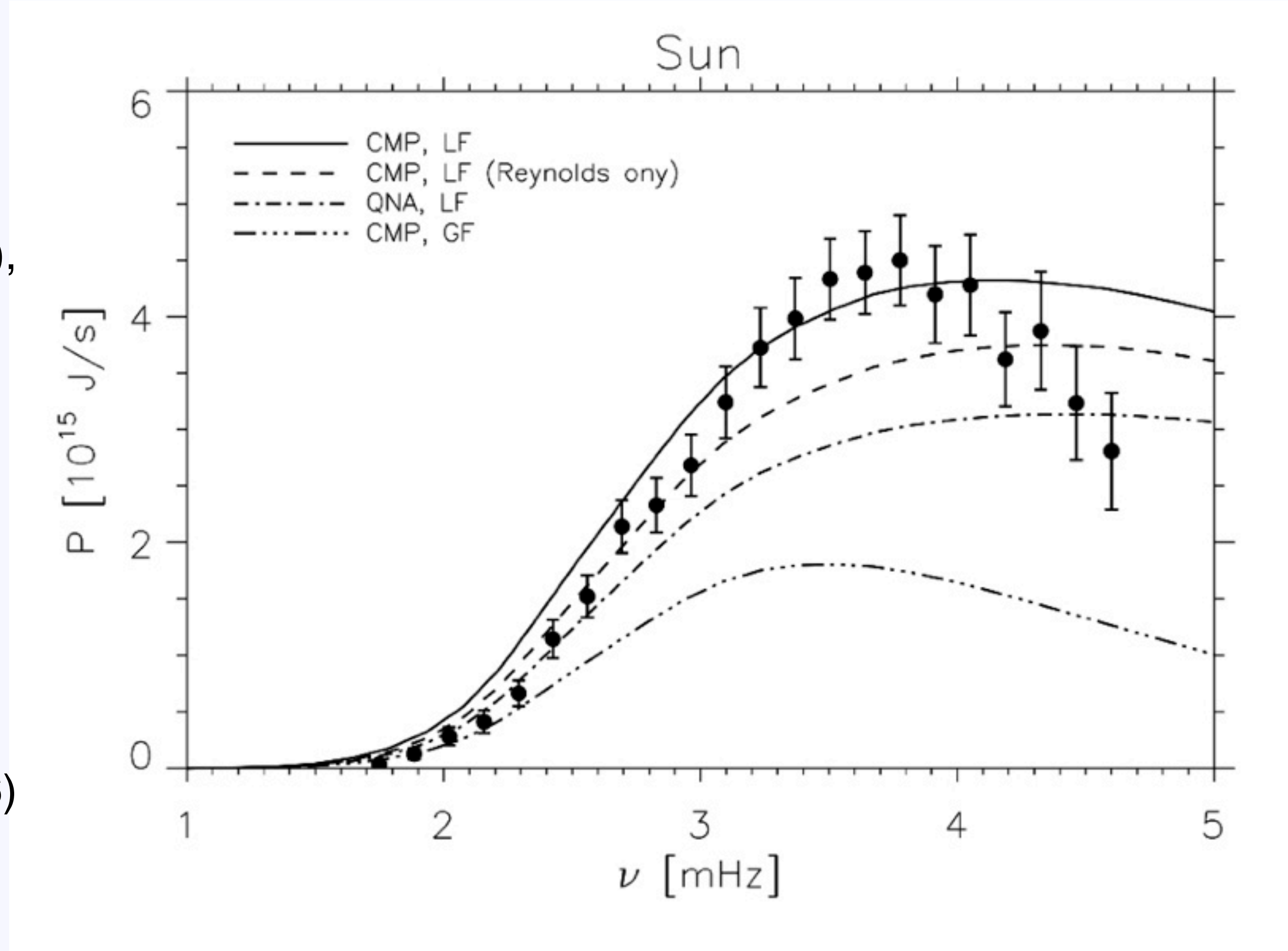


figure taken from Samadi et al. (2007), IAUS 239, pp. 349

...THANK YOU FOR YOUR TIME !